Four Games in Fifty Minutes

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Applied Mathematics

Real World

Interaction among
people
animals
companies
nations
genes
…

Assumptions

Math World

Game
players
rules
outcomes
preferences

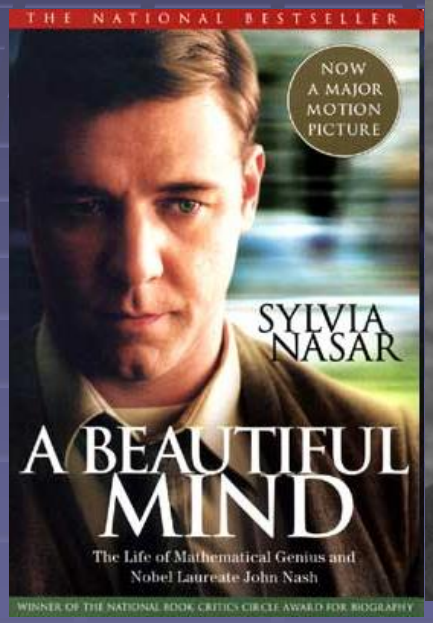
Techniques

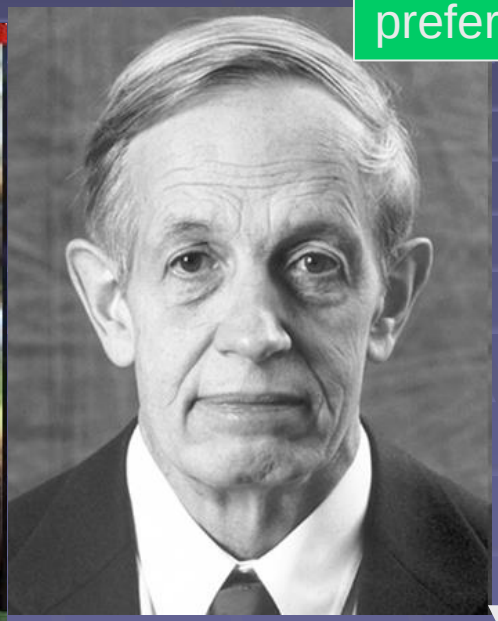
Solution

Interpretation

Prediction

Verification





Game 1: C or D

- Players: you and a randomly chosen opponent.
- Rules: secretly write your name and C or D on a slip of paper; hand me the paper.
- Outcomes: two randomly chosen pairs will receive the money indicated in the table.
- Preferences: there needs to be something left to model!

You choose	Opponent chooses	You receive
C	C	$3
D	C	$5
C	D	$0
D	D	$2

Analysis 1: C or D

- Assuming self-interested players,
  - D is dominant
  - D is prudential
  - (D, D) is the unique Nash equilibrium
  - (C, C) would be preferable
- For group-interested players,
  - Replace D with C in the above
- Real World
  - Nations Cooperate or Defect on reducing carbon emissions
  - Companies Collude or Don't on prices

You choose	Opponent chooses	You receive
C	C	$3
D	C	$5
C	D	$0
D	D	$2

Game 2: Repeated C or D

- Choose an opponent.
- Secretly write C or D on the My Choice/My Payoff slip of paper.
- Show each other your choices and write your own payoff from the table.
- I will roll a 6-sided die. If it lands on 1 through 5, repeat the previous two steps.
- If the die lands on 6, calculate your average payoff.
- Two randomly chosen pairs will receive their average payoffs.

You choose	Opponent chooses	You receive
C	C	$3
D	C	$5
C	D	$0
D	D	$2

Analysis 2: Repeated C or D

- For self-interested players,
  - No dominant strategy
  - Always choose D is prudential
  - Tit-for-Tat is a Nash equilibrium
  - Many other Nash equilibria

You choose	Opponent chooses	You receive
C	C	$3
D	C	$5
C	D	$0
D	D	$2

Game 3: Chocolate and Cash

- Write down how much (in dollars and cents) this chocolate is worth to you (that is, how much you would be willing to pay to purchase the chocolate and how much payment you would be willing to receive if you were to sell it). Do this now!
- Reveal to the group how much the chocolate is worth to you. Do this now!
- Two persons will be chosen to receive an inheritance of the chocolate and a certain amount of cash (between the average and maximum valuations of the chocolate). They will negotiate an allocation while everyone else writes down what they think would be a fair settlement. Do this now!
- We will compare the negotiated allocation with audience and spreadsheet suggestions.

Analysis 3: Chocolate and Cash

- Assume self-interested players with additive preferences.
- Consider the following fairness properties.
  - The division should depend only on possible improvements from no agreement.
  - A symmetric situation should result in an equal split division.
  - Rescaling and recentering preferences should not change the division.
  - Removing other possibilities should not change the division.
- John Nash showed that equal gains is the unique method satisfying the above fairness properties
- Real World
  - Inheritance
  - Labor contracts

Game 4: A Strange Auction

- Open ascending bid auction for a prize.
- The highest bidder wins the prize and pays her bid.
- The second highest bidder wins nothing and pays his bid.
- No one else pays.

Analysis 4: A Strange Auction

- Assume
  - I know what the prize is worth to me.
  - I do not know what the prize is worth to you, but I know the probability distribution from which your worth is derived.
- The unique symmetric Nash equilibrium
  - Bids increase with increasing worth
  - Some bids are greater than the worth
  - Yields a positive expected payoff that increases with increasing worth.
- Real Word
  - Animals competing for territory or a mate

Analysis 4: Some Details

- Suppose $f\ (v)$ is the probability density the prize is worth v to a player.
- Suppose $\beta(v)$ is the opponent's bid if the prize is worth v to him.
- If I value the prize at v and bid b, my expected payoff is

$$\pi(b) = \int_{\beta(u)<b} \big(v - \beta(u)\big) f(u) du - \int_{\beta(u)\geq b} b f(u) du$$

- Solve $\pi'\big(\beta(v)\big) = 0$ to obtain
- $\beta(v) = \int_0^v \frac{u\, f(u)}{1 - F(u)} du$

Thank you!

- Mathematical Modeling
- Game 1: C or D
- Game 2: Repeated C or D
- Game 3: Chocolate and Cash
- Game 4: A Strange Auction
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- Questions?